JOURNAL OF INFORMATION AND COMMUNICATION TECHNOLOGY
http://e-journal.uum.edu.my/index.php/jict

How to cite this article:
Hafeez, W., & Aziz, N. (2022). Bayesian two-sided complete group chain sampling plan for binomial distribution using beta prior through quality regions. *Journal of Information and Communication Technology, 21*(1), 51-69. https://doi.org/10.32890/jict2022.21.1.3

Bayesian Two-sided Complete Group Chain Sampling Plan for Binomial Distribution using Beta Prior through Quality Regions

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Received: 28/6/2020 Revised: 1/4/2021 Accepted: 6/7/2021 Published: 11/11/2021

ABSTRACT

Acceptance sampling is a technique for statistical quality assurance based on the inspection of a random sample to decide the lot disposition: accept or reject. Producer's risk and consumer's risk are inevitable in acceptance sampling. Most conventional plans only focus on minimizing the consumer's risk. This study focused on minimizing both producer's and consumer's risks through the quality region. Experts from available historical knowledge concurred that Bayesian is the best approach to make the correct decision. In this study, a Bayesian two-sided complete group chain sampling plan (BTSCGChSP) was proposed for the average probability of acceptance. The binomial distribution was used to derive the probability of lot acceptance, and the beta distribution was used as the prior distribution. For selected design parameters in BTSCGChSP, the acceptable quality level and limiting quality level were considered to estimate quality regions that were directly associated with producer's and consumer's risks, respectively. Four quality regions: (i) quality decision region , (ii) probabilistic quality region (PQR), (iii) limiting quality region, and (iv) indifference quality region, were evaluated. To compare with the existing Bayesian group chain sampling plan (BGChSP), operating characteristic curves were used for the same parameter values and probability of lot acceptance. The findings explained that BTSCGChSP provided a smaller proportion of defectives than BGChSP for the same probability of acceptance. If quality regions were found for the same values of consumer and producer risks, then the BTSCGChSP region would contain fewer defectives than in the BGChSP region. Therefore, for industrial practitioners, the proposed plan is a better substitute for existing BGChSP and other conventional plans.

Keywords: Acceptance sampling, Bayesian group chain, beta distribution, binomial distribution, quality region.

INTRODUCTION

Acceptance sampling is a key tool in quality control statistics. There are numerous steps involved in the acceptance of a product (Montgomery, 2020). In acceptance sampling, the product inspection is based on defective or non-defective. Acceptance sampling is one option for making a decision about a lot, and 100% inspection is another. As a result, the cost and time spent on inspection can be reduced because of acceptance sampling, only a limited number of samples are selected (Dobbah et al., 2018). Over the decades multiple acceptance sampling plans have been proposed. These include the single sampling plan (SSP), which was first introduced by Epstein (1954). Dodge (1955) improved the probability of lot acceptance by developing the chain sampling plan-1 (ChSP-1). He compares the previous sample to the current sample, and his decision is based on the cumulative results of both samples. In addition, Hald (1965) proposed SSP to reduce average costs.

Mughal et al. (2010) evaluated the design parameters for the group acceptance sampling plan (GASP). The sample size in GASP is divided into different equal number of groups based on the available number of testers ($n=r*g$). Many authors discussed various approaches to sampling plans. Most often they are risk-based sampling plans, with the plans based on consumer or producer risk as represented by the operating characteristic (OC). These plans do not explicitly take into account the previous history of similar lots submitted for inspection purposes. A Bayesian approach has been used by a number of researchers over the last century. This approach to sampling plan design considers previous lot quality history. Bayesian sampling plans necessitate the explicit specification of the distribution from lot to lot, which is referred to as prior distribution. Hafeez and Aziz (2019), Wang and Park (2020), Prajapati et al. (2020), Chen et al. (2021a), Chen et al. (2021b) and Hafeez et al. (2022) are a few recent studies that deal with the Bayesian approach. Hafeez and Aziz (2019) consider the quality variation for GChSP using the Bayesian approach. They used the binomial distribution with the beta distribution as a prior and devised a plan known as the Bayesian group chain sampling plan (BGChSP).

In the Bayesian framework, an SSP was modified with acceptable quality limit (AQL) and limiting quality level (LQL) using weighted Poisson distribution (Subramani & Haridoss, 2013; Subbiah & Latha, 2017) and Poisson distribution (Raju & Vidya, 2017). Latha and Suresh (2002) suggested a BChSP and used the gamma as a prior for performance measurements. Nirmala and Suresh (2018) developed a plan for Bayesian constrained repeating group sampling using quality regions. Statistical process control was used to control the variability of the products, control charts were applied to handle the variability of quantitative variable, and acceptance sampling was utilized for attributes (Aradea et al., 2020; Oakland & Oakland, 2018). The Bayesian approach was based on conditional probability, which depended on current and past information (Hald, 1965; Latha & Arivazhagan, 2015; Suresh & Latha, 2001; Suresh & Sangeetha, 2011; Zamzuri, Shabadin, & Ishak, 2019).

Under the acceptance sampling, this study proposes a new idea based on a BGChSP developed by Hafeez and Aziz (2019). The BGChSP was limited to the preceding lots and provided less protection to the consumer and producer. The objective of this study is to consider preceding as well as succeeding lots with the current lot to provide more protection to the consumer and producer. The suggested plan, Bayesian two-sided complete group chain sampling plan (BTSCGChSP), will perform better in handling the variability and acceptance criteria than BGChSP. From the addition of succeeding lots, the acceptance criteria will change and protect the current lot from both sides. If BGChSP considers four preceding lots $(i = 4)$, then the same approach in BTSCGChSP will consider four preceding and succeeding lots$(i = j = 4)$.To design BTSCGChSP, the indexed parameters are producer's risk(α),which is associated with AQL, and consumer's risk(β),which is associated with LQL for the specified values of shape parameter(s), number of groups(g),number of testers (r), and number of preceding and succeeding lots (r) and number of preceding and succeeding lots (i, j). Based on the prior shape parameter, numerical illustrations of the quality decision region (QDR), probabilistic quality region (PQR), limiting quality region (LQR), and indifference quality region (IQR) are also shown.

METHODOLOGY

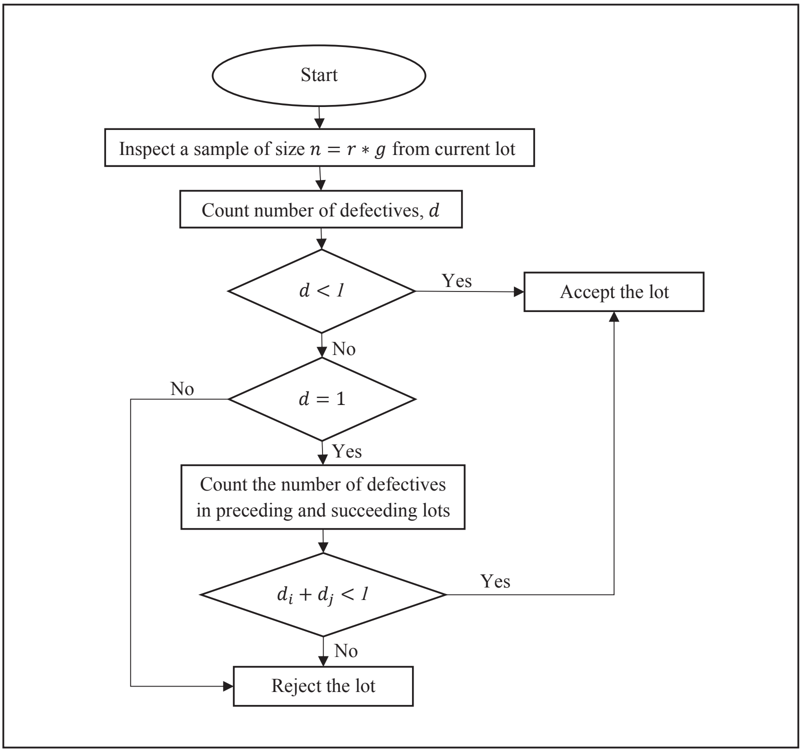
The procedure to perform BTSCGChSP) is based on the following steps:

- Select an ideal number of g groups for each lot and assign r items to each group, which is the sample size required $(n = g * r)$.
- Count the number of defectives, which is the sum of current d, preceding d_i, and succeeding d_j defectives.
- If $d = 0$ in the current sample, then accept the lot.
- Reject the lot if more than one defective is found in the current sample, i.e.,$(d > 1)$.
- If $d = 1$ in he current sample, but preceding i and succeeding j samples have no defectives$(d_i + d_j < 1)$, then accept the lot. However, reject the lot if preceding i and succeeding j samples have one or more defective in total, i.e.,$(d_i + d_j \geq 1)$.

All of the above steps can be summarized in a flow chart as shown in Figure 1.

Figure 1

Operating Procedure for BTSCGChSP.



For the number of preceding samples to be equal to the number of succeeding samples, BTSCGChSP is more useful for past and future predictions (Deva & Rebecca, 2012). For BTSCGChSP, the operating characteristic function $L(P)$ is:

$$L(P) = P_{0,(n)} + \left(P_{0,(n)}\right)^{i} P_{1,(n)} \left(P_{0,(n)}\right)^{j} \tag{1}$$

where $P_{0,(n)}$ and $P_{1,(n)}$ are the probability of non-defectives and defectives, respectively.
Now, the general expression of the probability of acceptance in group chain sampling for $i = j$ from Equation 1 is:

$$L(P) = P_{0,(r*g)} + P_{1,(r*g)} \left(P_{0,(r*g)}\right)^{2i} \tag{2}$$

The binomial distribution can be used to calculate the probability of zero and one defective products. (Montgomery, 2009; Teh et al., 2021). The defective and non-defective outcomes follow all four properties of a binomial experiment, which refers to the binomial distribution (Rosaiah & Kantam, 2005). It is valid because in the lot, all products follow the four properties of binomial random variable that are, all products are independent and identical. The probability of defective is constant and each product can be divided into two mutually exclusive outcomes that are either defective or non-defective. Therefore, the following likelihood function $L(d)$, can be used to estimate the probability of acceptance:

$$L(d) = \sum_{d=0}^{1} \binom{r * g}{d} p^{d}(1-p)^{r*g-d}, \tag{3}$$

where the probability of defectives is represented by p (the unknown parameter), d is the number of defectives, g is the number of groups, and r is the number of testers.

For zero and one defectives, after replacing $d = 0$ and $d = 1$ in Equation 3, we obtained:

$$P_0 = (1-p)^{r*g} \tag{4}$$

$$P_1 = (r * g)p(1-p)^{r*g-1} \tag{5}$$

Now the probability of lot acceptance for TSCGChSP after substituting Equations 4 and 5 in Equation 2 is:

$$L(p) = (1-p)^{rg} + rgp(1-p)^{rg-1}(1-p)^{2irg} \tag{6}$$

Beta distribution can be used as a suitable prior distribution, if the sample information has binomial distribution (Hafeez & Aziz, 2019; Latha & Arivazhagan, 2015). This defines that the parameter p follows beta distribution. The beta distribution, under the proposed sampling plan, has the following probability distribution function:

$$f(p) = \frac{1}{\beta(s,t)} p^{s-1}(1-p)^{t-1}, \tag{7}$$

where both $s, t > 0$ are shape parameters and the mean $\mu = \frac{s}{s+t}$.

Now, for BTSCGChSP based on the beta binomial distribution, the general equation for the average probability of acceptance is (Hafeez et al., 2022):

$$\bar{P} = \int_0^1 L(p)f(p)\,dp. \tag{8}$$

After substituting Equations 6 and 7 in Equation 8, are obtained:

$$\bar{P}=\int_0^1\left((1-p)^{rg}+rgp(1-p)^{rg-1}(1-p)^{2irg}\right)*\frac{1}{\beta(s,t)}p^{s-1}(1-p)^{t-1}\,dp \quad (9)$$

$$\bar{P}=\frac{1}{\beta(s,t)}\left[\beta(s,rg+t)+rg\beta(s+1,rg(2i+1)+t-1)\right] \quad (10)$$

$$\bar{P}=\frac{\Gamma(s+t)\Gamma(rg+t)}{\Gamma(t)\Gamma(rg+s+t)}+rg\frac{s\,\Gamma(s+t)\Gamma(rg(2i+1)+t-1)}{\Gamma(t)\,\Gamma(rg(2i+1)+s+t)} \quad (11)$$

Equation 11 is based on a mixture of beta and binomial distribution. After simplifying Equation 11, the following are obtained:

$s=1$:

$$\bar{P}=\frac{1-\mu}{rg\mu+1-\mu}+\frac{rg\mu(1-\mu)}{(rg\mu(2i+1)+1-\mu)(rg\mu(2i+1)+1-2\mu)}; \quad (12)$$

$s=2$:

$$\bar{P}=\frac{(2-\mu)(2-2\mu)}{(rg\mu+2-\mu)(rg\mu+2-2\mu)}+\frac{2rg\mu(2-\mu)(2-2\mu)}{(rg\mu(2i+1)+2-\mu)(rg\mu(2i+1)+2-2\mu)(rg\mu(2i+1)+2-3\mu)}; \quad (13)$$

$s=3$:

$$\bar{P}=\frac{(3-\mu)(3-2\mu)(3-3\mu)}{(rg\mu+3-\mu)(rg\mu+3-2\mu)(rg\mu+3-3\mu)}+\frac{3rg\mu(3-\mu)(3-2\mu)(3-3\mu)}{(rg\mu(2i+1)+3-\mu)(rg\mu(2i+1)+3-2\mu)(rg\mu(2i+1)+3-3\mu)(rg\mu(2i+1)+3-4\mu)}. \quad (14)$$

Newton's approximation was employed to estimate the quality regions of BTSCGChSP, where by reducing $\bar{P}$. average proportion of defective μ was used as the point of control. The outcomes in Table 1 were obtained from the above Equations 12 – 14. It can be observed that for the given values of the average probability of acceptance, the average product quality was obtained for BTSCGChSP.

Table 1

μ Values in BTSCGChSP for Specified Values of $\bar{P}$, s, g, r, and i.

s	g	r	i	0.99	0.95	0.90	0.5	0.25	0.1	0.05	0.01
1	1	2	1	0.0259	0.0656	0.1027	0.3916	0.6404	0.8382	0.9156	0.9825
		3	2	0.0129	0.0344	0.0561	0.2711	0.5160	0.7591	0.8689	0.9718
		4	3	0.0082	0.0226	0.0380	0.2100	0.4368	0.6976	0.8293	0.9620
	2	2	1	0.0128	0.0331	0.0530	0.2405	0.4677	0.7192	0.8429	0.9653
		3	2	0.0065	0.0173	0.0287	0.1564	0.3473	0.6114	0.7679	0.9450
		4	3	0.0041	0.0114	0.0193	0.1172	0.2793	0.5355	0.7084	0.9267

(continued)

s	g	r	i	0.99	0.95	0.90	0.5	0.25	0.1	0.05	0.01
	3	2	1	0.0085	0.0222	0.0357	0.1736	0.3685	0.6298	0.7809	0.9486
		3	2	0.0043	0.0116	0.0193	0.1099	0.2617	0.5118	0.6880	0.9197
		4	3	0.0027	0.0076	0.0129	0.0813	0.2053	0.4345	0.6182	0.8939
	4	2	1	0.0063	0.0167	0.0269	0.1359	0.3041	0.5603	0.7274	0.9326
		3	2	0.0032	0.0087	0.0145	0.0847	0.2099	0.4401	0.6231	0.8957
		4	3	0.0020	0.0057	0.0097	0.0622	0.1623	0.3656	0.5484	0.8633
2	1	2	1	0.0292	0.0717	0.1095	0.3746	0.5973	0.7949	0.8849	0.9740
		3	2	0.0145	0.0369	0.0585	0.2463	0.4494	0.6727	0.7965	0.9467
		4	3	0.0091	0.0240	0.0389	0.1862	0.3648	0.5845	0.7219	0.9166
	2	2	1	0.0144	0.0360	0.0559	0.2169	0.3928	0.6056	0.7377	0.9224
		3	2	0.0072	0.0186	0.0297	0.1357	0.2734	0.4673	0.6081	0.8546
		4	3	0.0046	0.0121	0.0197	0.1002	0.2128	0.3834	0.5186	0.7911
	3	2	1	0.0095	0.0240	0.0375	0.1524	0.2914	0.4835	0.6220	0.8616
		3	2	0.0048	0.0124	0.0199	0.0936	0.1961	0.3557	0.4863	0.7637
		4	3	0.0030	0.0081	0.0132	0.0685	0.1500	0.2842	0.4015	0.6832
	4	2	1	0.0071	0.0180	0.0282	0.1175	0.2314	0.4013	0.5350	0.8014
		3	2	0.0036	0.0093	0.0150	0.0715	0.1528	0.2868	0.4040	0.6851
		4	3	0.0023	0.0060	0.0099	0.0521	0.1158	0.2256	0.3270	0.5976
3	1	2	1	0.0307	0.0744	0.1126	0.3680	0.5773	0.769	0.8626	0.9659
		3	2	0.0151	0.0381	0.0596	0.2373	0.4241	0.6311	0.7523	0.9218
		4	3	0.0095	0.0246	0.0394	0.1782	0.3400	0.5364	0.6638	0.8718
	2	2	1	0.0151	0.0373	0.0572	0.2088	0.3654	0.5538	0.6765	0.8773
		3	2	0.0075	0.0192	0.0302	0.1290	0.2500	0.4146	0.5352	0.7730
		4	3	0.0048	0.0124	0.0199	0.0948	0.1932	0.3342	0.4442	0.6855
	3	2	1	0.0100	0.0249	0.0384	0.1456	0.2660	0.4275	0.5458	0.7792
		3	2	0.0050	0.0128	0.0202	0.0885	0.1769	0.3068	0.4109	0.6487
		4	3	0.0032	0.0083	0.0133	0.0646	0.1347	0.2418	0.3315	0.5541
	4	2	1	0.0075	0.0186	0.0288	0.1117	0.2090	0.3472	0.4552	0.6930
		3	2	0.0037	0.0096	0.0152	0.0673	0.1368	0.2432	0.3326	0.5549
		4	3	0.0024	0.0062	0.0100	0.0490	0.1034	0.1892	0.2640	0.4628

DESIGNING PLANS FOR GIVEN QUALITY LEVELS

For the specified values of producer's risk (α), and consumer's risk (β) in the selection of BTSCGChSP, Tables 1 and 2 are used by following these three steps:

- First for the given AQL $(1-\alpha)$ and LQL (β), calculate the operating ratio to construct a plan.
- From Table 2, find operating ratio value which is approximately equal to the desired columns of AQL and LQL for the fixed s, g, r, and i values located in Table 1.

- The values of s, g, r, and i can be found for the matching located value of the operating ratio.

Table 2

Operating Ratios for Specified Values of Design Parametersand s, g, r, i, α, and β.

s	g	r	i	$\alpha = 0.05$			$\alpha = 0.01$			$\alpha = 0.10$		
				$\beta = 0.10$	$\beta = 0.05$	$\beta = 0.01$	$\beta = 0.10$	$\beta = 0.05$	$\beta = 0.01$	$\beta = 0.10$	$\beta = 0.05$	$\beta = 0.01$
1	1	2	1	14.979	13.959	14.979	32.411	35.403	37.99	8.160	8.913	9.564
		3	2	28.272	25.279	28.272	58.897	67.415	75.397	13.519	15.474	17.307
		4	3	42.606	36.732	42.606	85.381	101.51	117.74	18.378	21.848	25.342
	2	2	1	29.119	25.427	29.119	56.261	65.938	75.513	13.578	15.914	18.225
		3	2	54.542	44.319	54.542	94.764	119.03	146.49	21.299	26.753	32.924
		4	3	81.548	62.335	81.548	131.23	173.59	227.09	27.76	36.719	48.037
	3	2	1	42.739	35.183	42.739	74.359	92.194	111.996	17.64	21.871	26.569
		3	2	79.33	59.341	79.33	119.605	160.784	214.944	26.542	35.681	47.7
		4	3	117.636	81.353	117.636	158.444	225.404	325.932	33.596	47.794	69.109
	4	2	1	55.89	43.597	55.89	88.373	114.746	147.101	20.803	27.011	34.627
		3	2	102.629	71.39	102.629	136.413	193.152	277.669	30.316	42.925	61.708
		4	3	151.36	96.139	151.36	180.945	271.401	427.29	37.55	56.322	88.673
2	1	2	1	13.588	12.344	13.588	27.235	30.316	33.371	7.258	8.08	8.894
		3	2	25.625	21.558	25.625	46.538	55.1	65.494	11.499	13.615	16.183
		4	3	38.216	30.1	38.216	64.349	79.486	100.921	15.019	18.552	23.555
	2	2	1	25.641	20.506	25.641	42.11	51.297	64.142	10.841	13.206	16.512
		3	2	45.932	32.683	45.932	64.935	84.492	118.744	15.747	20.489	28.795
		4	3	65.561	42.975	65.561	84.064	113.695	173.449	19.476	26.341	40.184
	3	2	1	35.862	25.887	35.862	50.717	65.24	90.379	12.894	16.586	22.977
		3	2	61.388	39.088	61.388	74.082	101.268	159.044	17.879	24.44	38.383
		4	3	84.556	49.698	84.556	93.954	132.744	225.848	21.566	30.469	51.84
	4	2	1	44.527	29.726	44.527	56.159	74.865	112.143	14.216	18.951	28.387
		3	2	73.36	43.261	73.36	80.279	113.092	191.773	19.169	27.004	45.792
		4	3	99.034	54.194	99.034	99.182	143.786	262.753	22.762	32.999	60.301
3	1	2	1	12.983	11.595	12.983	25.071	28.123	31.491	6.831	7.662	8.58
		3	2	24.189	19.74	24.189	41.736	49.751	60.965	10.595	12.63	15.477
		4	3	35.388	26.944	35.388	56.38	69.761	91.623	13.625	16.859	22.143
	2	2	1	23.549	18.159	23.549	36.745	44.889	58.212	9.682	11.828	15.338
		3	2	40.318	27.915	40.318	55.007	71.012	102.564	13.732	17.728	25.604
		4	3	55.335	35.852	55.335	70.089	93.158	143.781	16.798	22.326	34.459
	3	2	1	31.337	21.951	31.337	42.747	54.58	77.918	11.144	14.229	20.313
		3	2	50.813	32.186	50.813	61.441	82.279	129.894	15.191	20.343	32.116
		4	3	67.021	40.092	67.021	76.376	104.711	175.045	18.17	24.911	41.643
	4	2	1	37.194	24.435	37.194	46.305	60.724	92.43	12.038	15.787	24.029
		3	2	57.99	34.761	57.99	64.985	88.884	148.282	16.015	21.905	36.543
		4	3	75.013	42.788	75.013	79.584	111.022	194.639	18.931	26.41	46.3

Example 1: From Table 1, for $s = 1, g = 2, r = 3, i = 2$, and $\bar{P} = 0.50$, the corresponding value of IQL is $\mu_0 = 0.1564$ while for $s = 2, g = 3, r = 3$ and $i = 2$, the corresponding values of AQL and LQL are $\mu_1 = 0.0199$ and $\mu_2 = 0.4863$, respectively. It can be observed that as the values of s and g increase, the average quality of the product decreases.

Example 2: Suppose $\mu_1 = 0.0095$ and $\mu_2 = 0.18$. Then, the calculated operating ratio is 18.9474. The approximate equal value that is close to the calculated operating ratio from Table 2 is 18.951 with the corresponding parametric values: $s = 2, g = 4, r = 2, i = 1$ with $\alpha = 0.10$ and $\beta = 0.05$. The parallel values for AQL and LQL according the above parameters from Table 1, are $\mu_1 = 0.0282$ and $\mu_2 = 0.5350$, respectively.

DESIGNING QUALITY INTERVALS FOR BTSCGCHSP

Quality Decision Region

In QDR, the product is accepted with an engineer's quality average. Quality is reliably maintained up to μ_* LQL and a sudden decline in quality is expected. It is defined as $(\mu_1 < \mu < \mu_*)$ and denoted by $d_2 = \mu_* - \mu_1$, which is derived from the equation of the average probability of acceptance:

$$\bar{P}(\mu_1 < \mu < \mu_*) = \frac{\Gamma(s+t)\Gamma(rg+t)}{\Gamma(t)\Gamma(rg+s+t)} + rg\frac{s\,\Gamma(s+t)\Gamma(rg(2i+1)+t-1)}{\Gamma(t)\,\Gamma(rg(2i+1)+s+t)}$$

Therefore, beta is the prior distribution with mean $\mu = \frac{s}{s+t}$ as the approximate average quality of the product.

Probabilistic Quality Region

In PQR, the product is accepted with a minimum probability of 0.10 and a maximum probability of 0.95. The probabilistic quality region is defined as $(\mu_1 < \mu < \mu_2)$ and its range of PQR is denoted by $d_2 = \mu_2 - \mu_1$, this is derived from the average acceptance probability equation:

$$\bar{P}(\mu_1 < \mu < \mu_2) = \frac{\Gamma(s+t)\Gamma(rg+t)}{\Gamma(t)\Gamma(rg+s+t)} + rg\frac{s\,\Gamma(s+t)\Gamma(rg(2i+1)+t-1)}{\Gamma(t)\,\Gamma(rg(2i+1)+s+t)}$$

Limiting Quality Region

In LQR, the product will be accepted with a minimum probability of 0.1 and a maximum probability of 0.9. It is defined as $(\mu_* < \mu < \mu_2)$ and denoted by $d_2 = \mu_2 - \mu_*$, this is derived from the average acceptance probability equation:

$$\bar{P}(\mu_* < \mu < \mu_2) = \frac{\Gamma(s+t)\Gamma(rg+t)}{\Gamma(t)\Gamma(rg+s+t)} + rg\frac{s\,\Gamma(s+t)\Gamma(rg(2i+1)+t-1)}{\Gamma(t)\,\Gamma(rg(2i+1)+s+t)}$$

Indifference Quality Region

In IQR, the product is accepted with a minimum probability of 0.50 and a maximum probability of 0.9. It is defined as $(\mu_1 < \mu < \mu_0)$ and denoted by $d_2 = \mu_0 - \mu_1$, this is derived from the average acceptance probability equation:

$$\bar{P}(\mu_1 < \mu < \mu_0) = \frac{\Gamma(s+t)\Gamma(rg+t)}{\Gamma(t)\Gamma(rg+s+t)} + rg\frac{s\,\Gamma(s+t)\Gamma(rg(2i+1)+t-1)}{\Gamma(t)\,\Gamma(rg(2i+1)+s+t)}$$

SELECTION OF SAMPLING PLAN

In Table 3, the values of ranges of QDR(d_1),PQR(d_2),LQR (d_3),and IQR (d_0) are shown with corresponding design parameters s, g, r, and i, Operating ratios $T = \frac{\mu_* - \mu_1}{\mu_2 - \mu_1}$, $T_1 = \frac{\mu_* - \mu_1}{\mu_2 - \mu_*}$ and $T_2 = \frac{\mu_* - \mu_1}{\mu_0 - \mu_1}$ were defined and used to characterize the sampling plan. For any given values of QDR (d_1), PQR (d_2), LQR (d_3), and IQR (d_0), the operating ratio $T = \frac{d_1}{d_2}$, $T_1 = \frac{d_1}{d_3}$ and $T_2 = \frac{d_1}{d_0}$ could be found. The authors were required to find the value corresponding to the design parameters s, g, r, and i, which were approximately equal to the specified ratio under the columns T, T_1, and T_2 in Table 3. The parameters of BTSCGChSP can be identified from this ratio.

Table 3

Selected Values of QDR, PQR, LQR, IQR, and Operating Characteristic Ratio for Specified Values of $s, g, r,$ *and* i.

s	g	r	i	μ_1	μ_*	μ_0	μ_2	d_1	d_2	d_3	d_0	T	T_1	T_2
1	1	2	1	0.0656	0.1027	0.3916	0.8382	0.0371	0.7726	0.7355	0.326	0.0481	0.0505	0.1139
		3	2	0.0344	0.0561	0.2711	0.7591	0.0218	0.7247	0.703	0.2367	0.03	0.031	0.092
		4	3	0.0226	0.038	0.21	0.6976	0.0154	0.675	0.6596	0.1875	0.0228	0.0233	0.082
	2	2	1	0.0331	0.053	0.2405	0.7192	0.0198	0.686	0.6662	0.2074	0.0289	0.0297	0.0956
		3	2	0.0173	0.0287	0.1564	0.6114	0.0114	0.594	0.5827	0.1391	0.0192	0.0195	0.0818
		4	3	0.0114	0.0193	0.1172	0.5355	0.0079	0.5242	0.5162	0.1059	0.0151	0.0154	0.0749
	3	2	1	0.0222	0.0357	0.1736	0.6298	0.0135	0.6076	0.5941	0.1515	0.0222	0.0227	0.0892
		3	2	0.0116	0.0193	0.1099	0.5118	0.0077	0.5002	0.4925	0.0983	0.0154	0.0156	0.0782
		4	3	0.0076	0.0129	0.0813	0.4345	0.0053	0.4269	0.4216	0.0737	0.0125	0.0127	0.0724
	4	2	1	0.0167	0.0269	0.1359	0.5603	0.0102	0.5436	0.5333	0.1192	0.0188	0.0192	0.086
		3	2	0.0087	0.0145	0.0847	0.4401	0.0058	0.4313	0.4255	0.076	0.0134	0.0136	0.0762
		4	3	0.0057	0.0097	0.0622	0.3656	0.004	0.3599	0.3559	0.0565	0.0112	0.0113	0.0713
2	1	2	1	0.0717	0.1095	0.3746	0.7949	0.0378	0.7232	0.6854	0.303	0.0523	0.0552	0.1249
		3	2	0.0369	0.0585	0.2463	0.6727	0.0216	0.6358	0.6142	0.2093	0.0339	0.0351	0.103
		4	3	0.024	0.0389	0.1862	0.5845	0.0149	0.5605	0.5455	0.1623	0.0266	0.0274	0.092
	2	2	1	0.036	0.0559	0.2169	0.6056	0.0199	0.5696	0.5497	0.1809	0.0349	0.0362	0.1099
		3	2	0.0186	0.0297	0.1357	0.4673	0.0111	0.4487	0.4376	0.1171	0.0247	0.0253	0.0945
		4	3	0.0121	0.0197	0.1002	0.3834	0.0076	0.3714	0.3638	0.0882	0.0205	0.021	0.0864

(continued)

s	g	r	i	μ_1	μ_*	μ_0	μ_2	d_1	d_2	d_3	d_0	T	T_1	T_2
	3	2	1	0.024	0.0375	0.1524	0.4835	0.0135	0.4595	0.446	0.1284	0.0293	0.0302	0.1049
		3	2	0.0124	0.0199	0.0936	0.3557	0.0075	0.3433	0.3358	0.0812	0.0217	0.0222	0.0918
		4	3	0.0081	0.0132	0.0685	0.2842	0.0051	0.2761	0.271	0.0604	0.0185	0.0188	0.0844
	4	2	1	0.018	0.0282	0.1175	0.4013	0.0102	0.3833	0.3731	0.0995	0.0267	0.0274	0.1028
		3	2	0.0093	0.015	0.0715	0.2868	0.0056	0.2774	0.2718	0.0621	0.0203	0.0207	0.0905
		4	3	0.006	0.0099	0.0521	0.2256	0.0039	0.2195	0.2157	0.046	0.0177	0.018	0.0842
3	1	2	1	0.0744	0.1126	0.368	0.769	0.0382	0.6946	0.6564	0.2936	0.055	0.0582	0.1301
		3	2	0.0381	0.0596	0.2373	0.6311	0.0215	0.593	0.5715	0.1992	0.0362	0.0375	0.1077
		4	3	0.0246	0.0394	0.1782	0.5364	0.0147	0.5118	0.4971	0.1535	0.0288	0.0296	0.096
	2	2	1	0.0373	0.0572	0.2088	0.5538	0.0199	0.5165	0.4966	0.1716	0.0386	0.0402	0.1162
		3	2	0.0192	0.0302	0.129	0.4146	0.011	0.3954	0.3844	0.1098	0.0279	0.0287	0.1003
		4	3	0.0124	0.0199	0.0948	0.3342	0.0075	0.3218	0.3143	0.0824	0.0233	0.0239	0.0911
	3	2	1	0.0249	0.0384	0.1456	0.4275	0.0135	0.4026	0.3891	0.1207	0.0335	0.0347	0.1118
		3	2	0.0128	0.0202	0.0885	0.3068	0.0074	0.2941	0.2866	0.0757	0.0253	0.0259	0.0981
		4	3	0.0083	0.0133	0.0646	0.2418	0.005	0.2335	0.2285	0.0563	0.0216	0.0221	0.0895
	4	2	1	0.0186	0.0288	0.1117	0.3472	0.0102	0.3285	0.3183	0.0931	0.0311	0.0321	0.1096
		3	2	0.0096	0.0152	0.0673	0.2432	0.0056	0.2336	0.228	0.0578	0.024	0.0246	0.0972
		4	3	0.0062	0.01	0.049	0.1892	0.0038	0.1831	0.1792	0.0428	0.0209	0.0213	0.0894

NUMERICAL EXAMPLES

For specified QDR and PQR

Let the percentage of defectives in QDR and PQR be 0.5 percent and 9 percent, respectively; then, the estimated operating ratio is $T = 0.0556$. From Table 3, it can be determined whether the value of T was approximately equal to the specified ratio. The value was found to be $T = 0.055$, with the parallel values of design parameters $s = 3$, $g = 1$, $r = 2$, and $i = 1$. For this operating ratio, QDR $d_1 = 0.0382$ and PQR $d_2 = 0.6946$. Therefore, the required design parameters were $s = 3$, $g = 1$, $r = 2$, and $i = 1$, with $\mu_1 = 0.0744$, $\mu_* = 0.1126$, and $\mu_2 = 0.7690$.

For specified QDR and LQR

Let the percentage of defectives in QDR and LQR be 0.5 percent and 10 percent, respectively; then, the calculated operating ratio is $T_1 = 0.05$. From Table 3, it can be determined whether the value of T_1 was equal to or less than the specified ratio. The value was found to be $T_1 = 0.0402$, with parallel design parameters values $s = 3$, $g = 2$, $r = 2$, and $i = 1$. For this operating ratio, QDR $d_1 = 0.0199$ and LQR $d_3 = 0.4966$. Therefore, the required design $s = 3$, $g = 2$, $r = 2$, and $i = 1$, with $\mu_1 = 0.0373$, $\mu_* = 0.0572$, and $\mu_2 = 0.5538$.

For specified QDR and IQR

Let the percentage of defectives in QDR and IQR be 1 percent and 10 percent, respectively; then $d_1 = 0.01$, $d_0 = 0.10$ and the computed operating ratio is $T_2 = 0.1$. From Table 3, it can be determined whether the value T_2 of was equal to or less than the specified ratio. The value was found to be $T_2 = 0.0981$, with the parallel design parameters values $s = 3$, $g = 3$, $r = 3$, and $i = 2$. For this operating ratio, QDR $d_1 = 0.0074$ and IQR $d_0 = 0.0757$. Therefore, the required design parameters were $s = 3$, $g = 3$, $r = 3$, and $i = 2$, with $\mu_1 = 0.0128$, $\mu_* = 0.0202$, and $\mu_0 = 0.0885$.

GRAPHS AND DISCUSSION

Figure 2 represents the operating characteristic (OC) curves for BTSCGChSP for the specified values of shape parameter $s = 2$, the number of testers $r = 2$, $i = 1$, and for different values of $g = 1, 2, 3, 4$. It can be observed that the OC curve became more ideal as the number of groups increased.

Figure 2

OC Curves for BTSCGChSP for $g = 1,\ 2,\ 3,\ 4$

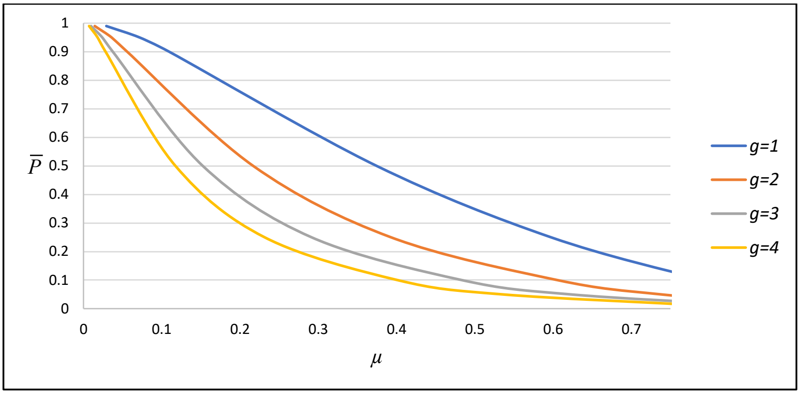
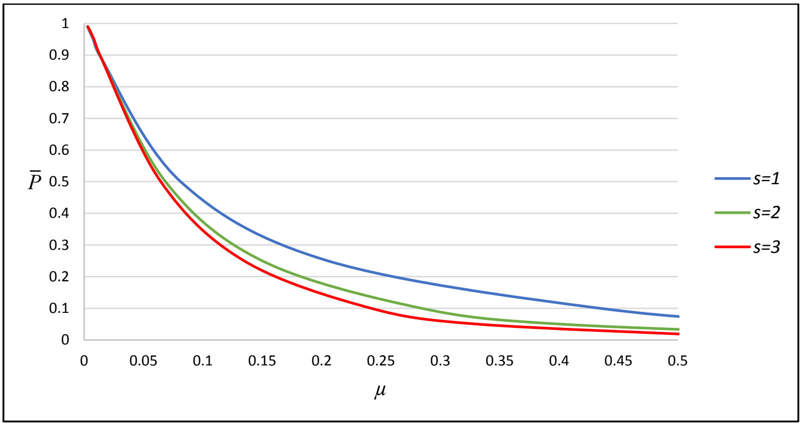


Figure 3 represents the OC curves for BTSGChSP for fix number of groups $g = 3$, number of testers $r = 4$, number of preceding lots $i = 3$, and for shape parameters values $s = 1, 2, 3$. As the values of the shape parameter increased, the OC curve became more ideal.

Figure 3

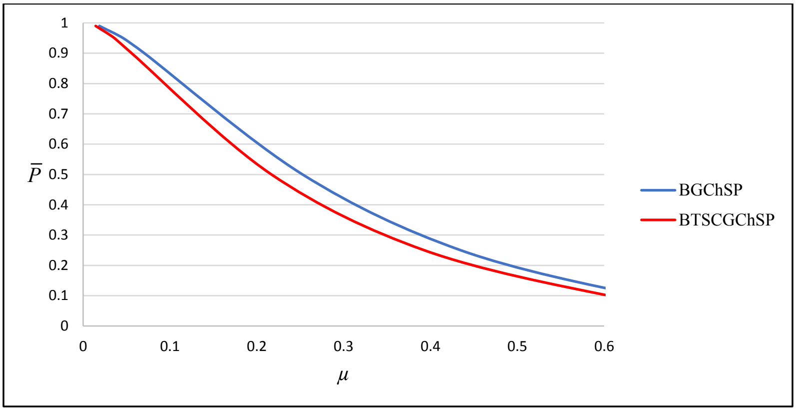
OC Curves for BTSCGChSP for



For the comparison study, the OC curves for the proposed BTSCGChSP and the existing BGChSP proposed by Hafeez and Aziz (2019) are represented. BGChSP only considers preceding lots while BTSCGChSP considers both preceding and succeeding lots. For both plans with the same values of shape parameter $s = 2$, number of testers $r = 2$, number of groups $g = 2$, and for $i = 1$, the OC curves are as presented in Figure 4.

Figure 4

Comparison between OC Curves of BTSCGChSP and BGChSP.



By comparing both curves, it can be observed that BTSCGChSP performed better than BGChSP. For the same design parameters, BTSCGChSP provided a lower proportion of defectives than existing BGChSP.

CONCLUSION

Acceptance sampling plans are becoming more popular in the industry as a way to ensure that a product or process meets a higher quality standard. As a result, quality regions may be helpful in outlining product quality and planning quality control arrangements for industrial applications. Because the proposed BTSCGChSP is less expensive, it will be beneficial to quality assurance practitioners. The main advantage of the proposed plan is that the future is more concerned because of succeeding lots than in the past. From this study, it is concluded that for same probability of acceptance and other parameter values, BTSCGChSP provides a smaller proportion of defectives than existing BGChSP. Furthermore, it is shown that as the values of design parameters increase, the OC curve approaches the ideal OC curve. The proposed work may be a useful addition to the literature and a helpful tool for quality engineers of internal control practitioners.

ACKNOWLEDGMENT

This work was supported by the Ministry of Higher Education (MoHE) of Malaysia through Fundamental Research Grant Scheme (FRGS/1/2020/STG06/UUM/02/2).

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